

# Optimal Inflation Target: A Simple Game-Theoretic Approach

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## Abstract

In an inflation-targeting regime, we study the strategic interaction between a continuum of anonymous market participants and the central bank (CB), modeled as a long-run player. A sufficiently patient CB can implement its preferred equilibrium. To do this, the CB does not need to play the tightest monetary policy all the time.

**Keywords:** games, monetary policy, inflation target, optimal target, central banks.

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# 1 Introduction

Inflation targeting has been used for more than a decade by a significant number of countries. Here, we develop a simple game-theoretic model in which the monetary authority, traditionally the country's central bank (CB), publicly announces a target for the future inflation, and then plays an infinitely repeated game with a continuum of anonymous myopic market participants. In this repeated game, the CB is a long-run player.

We find that the CB can always implement its preferred equilibrium in the repeated game. The policy maker does not need to play the tightest monetary policy all the time to achieve this equilibrium. The intensity of cooperation by the CB should be only sufficiently large to make market participants' best responses be their part in the prescribed equilibrium.<sup>1</sup> There is an inflation bias in the choice of the target. We discuss possible solutions.

We assume that the CB wants to decrease inflation as much as possible, but it also has a short-run incentive to run a loose monetary policy. Independently on its own action, the policy maker always wants to obtain cooperation from the market agents. In order to get this, the CB needs to establish credibility in the sense that market participants must believe that inflation will be sufficiently low in order to cooperate.

By choosing a target value for future inflation, the CB signals to the market the level that inflation should be at. Hence, at first glance, a low target is desired. Under a relatively more ambitious (i.e. lower) target, inflation may in fact become lower,

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<sup>1</sup>Woodford (1999) has a similar conclusion. In his words, "...some commentators have proposed that U.S. monetary policy has been so successful at inflation stabilization in the 1990s, despite relatively little change in the funds rate for years at a time, because 'the bond market does the Fed's work for it,' responding to disturbances in the way needed to keep inflation stable without the need for large policy adjustments by the Fed. This is exactly what a good policy regime should look like..."

provided that such a target is credible. However, under a smaller target, a defection by the CB is more costly to market agents who trusted in the announced policy than it would be under a relatively larger target. This suggests that the CB may obtain more cooperation when the target is larger (since market agents have less to lose). With more cooperation by market participants the inflation will decrease. Therefore, it is not clear *a priori* what is the optimal target.

## 1.1 Brief Literature Review

Inflation targeting was studied by Svensson (1996) and (1997), Mishkin and Schmidt-Hebbel (2001), Clarida *et al* (1999), Giannone and Woodford (2003), and Woodford (2003). Mishkin (2004) brings the particulars of inflation targeting in emerging markets. Fudenberg *et al* (1990) introduced the study of repeated games with short-run and long-run players.<sup>2</sup> Applications of game theory to monetary policy were made, among others, by Barro and Gordon (1983) and (1986), Barro (1986), and Persson and Tabellini (1995). Observe that, in contrast with many of these papers, we are not using incomplete information in our framework. Next section describes the game and section 3 calculates the best equilibrium for the CB. Section 4 discusses the inflation bias.

## 2 The Game

At the beginning of the game, the CB chooses the next inflation target, denoted  $\pi^* \in [0, 1]$ , and normalized to be inside the unit interval. The target choice defines stage-game payoffs of an infinitely repeated game, namely  $\pi_L$ ,  $\pi_M$ ,  $\pi_H$ ,  $c$  and  $g$ . In this repeated game, the monetary authority is a long-run player, named player 1. Player 2 is not strategic; it only summarizes the aggregate behavior of a continuum of

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<sup>2</sup>Alternative interpretations of short-run and long-run player models, as well as the interpretation of the anonymity assumption of short-run players are discussed in Mailath and Samuelson (2006).

anonymous and myopic market agents. Describing market participants as anonymous short-run players is a convenient way of modeling agents that cannot be individually punished by the CB because it is too costly to perfectly observe their individual behavior, or the CB cannot legally punish a group of market agents.<sup>3</sup>

## 2.1 Actions

Each market agent chooses a pure action and her payoff depends only on her own action and on the action of the CB,<sup>4</sup> while the payoff of the CB depends on its own action and on the aggregate behavior of all market players.

In the stage-game, the CB chooses a mixture of what we call cooperation ( $C$ ) and defection ( $D$ ). Formally,  $A_1 = \{C, D\}$  represents the set of pure actions available to the policy maker. Mixed actions of the CB are characterized by the intensity of cooperation, that is, the probability of  $C$ , denoted by  $x \in [0, 1]$ . Each market agent  $j \in (0, 1)$  chooses a pure action, denoted by  $a_j$ , in the set  $\{C, D\}$ . All actions are observable, including distributions of mixed actions.

## 2.2 Utility of the Players

In each round  $t$ , let  $u_1^t$  denote the payoff of player 1 in period  $t$ . Whenever the context is clear, we will omit the time superscript, and player 1's payoff will be denoted by  $u_1$ . The utility function of the CB, denoted by  $U_1$ , is defined as:

$$U_1 = (1 - \delta) \sum_{t=0}^{+\infty} \delta^t u_1^t,$$

where  $\delta \in (0, 1)$  represents the CB's discount factor.

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<sup>3</sup>Alternatively, we can imagine that in each round a market participant is randomly selected to act as player 2. See chapter 2 of Samuelson and Mailath (2006) for more details.

<sup>4</sup>Allowing the payoff of a market participant to also depend on the aggregate behavior of other market agents does not bring any new issue. See chapter 2 of Mailath and Samuelson (2006). Allowing that each market participant plays a mixed action can only increase the technicalities of the model.

On the other hand, market agents are myopic. They play stage-game best responses in every round. The aggregate behavior of a continuum of anonymous market agents is referred to as the mixed action of player 2. This is not a strategic player since we do not assume any coordination among market agents.

### 2.2.1 Stage-Game Payoffs

Figure 1 brings the stage-game payoffs. We assume that  $\pi_H > \pi_M > \pi_L > 0$  represent high ( $H$ ), medium ( $M$ ) and low ( $L$ ) levels of inflation, respectively. The cost for the CB to play a tight monetary policy is denoted  $c$ , and the magnitude of each market player's maximum payoff is represented by  $g > 0$ . We also assume that all these values are smooth functions of the target such that for any  $\pi^* \in [0, 1]$ :<sup>5</sup>

$$\frac{\partial \pi_L}{\partial \pi^*} > \frac{\partial \pi_M}{\partial \pi^*} > \frac{\partial \pi_H}{\partial \pi^*} > 0, \quad \frac{\partial c}{\partial \pi^*} < 0, \quad \frac{\partial g}{\partial \pi^*} < 0, \quad \text{and}$$

$$c \in \left( \text{Max} \{ \pi_H - \pi_M, \pi_M - \pi_L \}, 2\pi_H - \pi_M - \pi_L \right)$$

All inflation levels  $\pi_\omega$ , with  $\omega \in \{L, M, H\}$ , are increasing in the inflation target. The lower the inflation level is, the more sensitive to target changes the inflation level will be. Formally,  $\frac{\partial \pi_L}{\partial \pi^*} > \frac{\partial \pi_M}{\partial \pi^*} > \frac{\partial \pi_H}{\partial \pi^*}$ .

On the other hand, the cost  $c$  of implementing a tight monetary policy is decreasing in the target. The lower bound for  $c$  makes the stage-game be such that the CB always prefers to play  $D$ , no matter what proportion of market agents play  $C$ . In other words, we assume that the cost for the CB of reducing inflation directly is always larger than its direct benefit. Because the stage-game is played repeatedly, reputation effects provide a reason for the CB to play a mixed action in every round. If the cost  $c$  were sufficiently small, the outcome  $(C, C)$  would be played in every round.

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<sup>5</sup>Note that  $\pi_H > \pi_M > \pi_L$  implies that  $2\pi_H - \pi_M - \pi_L > \pi_H - \pi_M$ , and that  $2\pi_H - \pi_M - \pi_L > \pi_M - \pi_L$ .

		Market Agent	
		C	D
Central Bank	C	- $\pi_L - c$ ,    + $g$	- $\pi_M - c$ ,    - $g$
	D	- $\pi_M$ ,        - $g$	- $\pi_H$ ,        + $g$

Figure 1: Stage-game payoffs.

As we will see ahead, the upper bound on  $c$  is necessary for the implementation of the repeated game Nash equilibrium.

### 2.2.2 Payoffs of Mixed Actions

Any mixed action  $\alpha_1$  of the CB can be characterized by  $x \in [0, 1]$ , the intensity that player 1 chooses  $C$ . This is consistent with the interpretation of  $x$  as an increasing function of the interest rate set by the monetary authority. Similarly, player 2's mixed action  $\alpha_2$  is characterized by  $y \in [0, 1]$ , the proportion of market agents playing  $C$ .<sup>6</sup> With this notation, stage-game payoffs associated with the action profile  $(x, y)$  are:

$$u_1(x, y) = x [y (2\pi_M - \pi_H - \pi_L) + \pi_H - \pi_M - c] - [y\pi_M + (1 - y)\pi_H] \quad (1)$$

$$u_2(x, y) = 2g [2x - 1] y + g(1 - 2x) \quad (2)$$

### 2.2.3 Economic Intuition of Payoffs

Now, the intuition behind stage-game payoffs will be described. Cooperation by player 1 means that the CB is playing a tight monetary policy. We are assuming

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<sup>6</sup>When the alternative interpretation of footnote 5 is used for player 2,  $y$  is the probability that the randomly chosen agent plays  $C$ .

that other sectors in the government do not mitigate CB's actions by their own choices of policies. Cooperation by player 2 means that all market agents act in a way that is consistent with the announced inflation target.

Whenever player 2 cooperates, the CB obtains  $-\pi_L - c$  by running the tightest monetary policy (playing  $C$ ), and  $-\pi_M$  by defecting. Observe that playing  $D$  is a dominant strategy for player 1, meaning that the CB is always tempted to run a loose monetary policy. By assumption,  $-\pi_L - c > -\pi_M - c$ , and  $-\pi_M > -\pi_H$ . Hence, player 1 always benefits if all market participants play  $C$ .

For market players, the payoff  $g$  decreases with the target. When all players coordinate their efforts to fight inflation, the outcome is  $(C, C)$ , and then, inflation is lower the smaller the target is. At the same time, we may observe that the effort to play  $C$  may depend on  $\pi^*$ . If the target is relatively smaller, playing  $C$  may require the CB to choose a relatively higher interest rate. Technically speaking, we model this by assuming that the function  $\pi^* \mapsto c(\pi^*)$  is decreasing and convex, with  $\frac{\partial c}{\partial \pi^*}(0)$  being sufficiently low (negative, with a large absolute value).

For markets participants, the goal is to coordinate their action with the CB. Their payoffs in  $(C, C)$  and in  $(D, D)$  are decreasing in the target. On the other hand, if a market player misses the CB's action by playing  $C$  when player 1 plays  $D$  or vice-versa, she obtains only  $-g$ , which is the lowest feasible payoff. This payoff is an increasing function of the inflation target. The more ambitious the target is (lower  $\pi^*$ ), the more costly miscoordination becomes to market players. The intuition behind this is that the lower the inflation target is, the stronger will be the implicit contract between the CB and market players. Therefore, the lower  $\pi^*$  is, the larger is the degree of the mistake that market agents are doing by betting on the contrary action that the CB implements.



## 2.3 Inflation

The inflation in a given round  $t$ , denoted by  $\pi^t$ , is determined by the outcome  $(x, y)$  of the stage-game at period  $t$ . Formally:

$$\pi^t = xy \cdot \pi_L + x(1 - y) \cdot \pi_M + (1 - x)y \cdot \pi_M + (1 - x)(1 - y) \cdot \pi_H$$

The inflation accumulated over the repeated game, denoted by  $\Pi$ , is the discounted average of the inflations in each round. Mathematically:

$$\Pi = (1 - \delta) \sum_{t=0}^{+\infty} \delta^t \pi^t$$

## 3 Equilibrium Analysis

Recall that a strategy  $\sigma_1$  for the CB in a repeated game is a function assigning a mixed action  $x \in [0, 1]$  to any past history of play. Let  $\sigma_2$  denote the profile of strategies of all short-run players. A strategy profile  $\sigma = (\sigma_1, \sigma_2)$  for the repeated game with a long-run and a continuum of short-run players is a Nash equilibrium if and only if:

- Player's 1 strategy  $\sigma_1$  maximizes  $U_1(\sigma_1, \sigma_2)$  over all repeated game strategies, and
- At any history of play that is reached with positive probability under the profile  $\sigma$ , every market player plays a best response against the prescribed action of player 1 in  $\sigma$  at that history.<sup>7</sup>

### 3.1 Nash Equilibria of the Stage-Game

Every market agent prefers to cooperate ( $C$ ) whenever the CB puts sufficiently high weight on cooperation, that is,  $x > 1/2$ . If  $x = 1/2$ , market players are indifferent

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<sup>7</sup>Recall that the actions of other market players do not affect the payoff of a specific market agent.

between  $C$  and  $D$ . If  $x < 1/2$ , the best response for any market participant is defection ( $D$ ). Notice that the threshold  $x = 1/2$  is independent of the target  $\pi^*$ .

The set of all Nash equilibria of the stage-game is given by all profiles in which the CB puts sufficient weight on  $D$  and all market players play  $D$ . Mathematically:

$$\{(x, 0) \mid x \leq 1/2\}$$

Observe that the CB can induce every market agent to cooperate by choosing  $x$  just above  $1/2$ . Next, we will describe the maxmin payoff of the long-run player.

### 3.2 Market Best Response and CB's Maxmin Payoff

Recall that player 2 represents the aggregate behavior of all market agents. The best response of player 2 is given by a correspondence from the set of player 1's mixed actions, denoted  $\Delta A_1$ , to the set  $\Delta A_2$  of player 2's mixed actions. The graph of player 2's best response correspondence is denoted  $B \subset \Delta A_1 \times \Delta A_2$ .

Define player 1's maxmin payoff against a short-lived player, denoted by  $\bar{v}_1$ , with:

$$\bar{v}_1 = \sup_{\alpha \in B} \min_{a_1 \in \text{supp}(\alpha_1)} u_1(a_1, \alpha_2),$$

where  $\text{supp}(\alpha_1)$  denotes the support of the mixed action  $\alpha_1$ . Fudenberg *et al* (1990) proves that  $\bar{v}_1$  is the maximal utility that the long-run player may obtain in any Nash equilibrium of the repeated game. Next, we will describe the equilibrium in which the policy maker has the highest possible utility.

### 3.3 Preferred Equilibrium of the CB

We propose an equilibrium where in every round the action profile  $(1/2, 1)$  is played. Deviations by market agents are ignored and any deviation by the CB, even if many market players also deviate, triggers perpetual play of mutual defection, i.e. perpetual play of the action profile  $(0, 0)$ . Deviations by any player, or by any collection of

players, in any stage of the punishment phase do not change the future prescribed play of the punishment phase.

The best payoff for player 1 occurs when  $y = 1$  because, by assumption,  $-\pi_L - c > -\pi_M - c$  and  $-\pi_M > -\pi_H$ . At  $(1/2, 1)$ , this payoff becomes:

$$u_1(1/2, 1) = \frac{-1}{2} (\pi_M + \pi_L + c) \quad (3)$$

If this value is obtained in every period, the CB's utility also becomes  $U_1 = u_1(1/2, 1)$ . This is the best that the monetary authority could hope for because it is equal to its maxmin payoff, that is:

$$U_1 = u_1(1/2, 1) = \bar{v}_1$$

From equation (3) we can see that, for any fixed target  $\pi^* \in [0, 1]$ , the value  $U_1 = u_1(1/2, 1)$  is decreasing in  $\pi_M$ ,  $\pi_L$  and  $c$ . How does the CB's utility depends on the inflation target? In (3), taking the derivative of both sides with respect to the target  $\pi^*$ :

$$\frac{\partial u_1(1/2, 1)}{\partial \pi^*} = \frac{-1}{2} \left( \frac{\partial \pi_M}{\partial \pi^*} + \frac{\partial \pi_L}{\partial \pi^*} + \frac{\partial c}{\partial \pi^*} \right)$$

The CB can maximize its utility by choosing a target  $\pi^*$  that solves the problem:

$$\underset{\pi^* \in [0, 1]}{Max} \left\{ \frac{-1}{2} (\pi_M + \pi_L + c) \right\} \quad (4)$$

The first order condition of the CB's maximization problem is:

$$\frac{\partial \pi_M}{\partial \pi^*} + \frac{\partial \pi_L}{\partial \pi^*} = \frac{\partial(-c)}{\partial \pi^*} \quad (5)$$

Because the target is inside a compact set,  $\pi^* \in [0, 1]$ , the maximization problem (4) always has a solution. Uniqueness of the solution depends on the behavior of the functions  $\frac{\partial \pi_M}{\partial \pi^*}$ ,  $\frac{\partial \pi_L}{\partial \pi^*}$ , and  $\frac{\partial c}{\partial \pi^*}$ .

### 3.3.1 Incentives

Why the perpetual play of  $(1/2, 1)$  is a Nash equilibrium, in fact a subgame perfect equilibrium, of the repeated game? Well, each market agent is playing a best response in every period. For player 1, a deviation triggers a Nash reversion to the path of mutual defection in every round. The outcome  $(D, D)$  is a Nash equilibrium of the stage-game. Thus, the profile  $(D, D)$  in every round is a Nash equilibrium of the repeated game. This takes care of the incentives in the punishment path.

Now, we must verify if the CB prefers the proposed path of perpetual outcome  $(1/2, 1)$ , which implies obtaining a utility of  $U_1 = u_1(1/2, 1)$ , rather than obtaining  $(1 - \delta)(-\pi_M)$  in the first round followed by  $-\pi_H$  in every round thereafter. This is the case when:

$$\frac{-1}{2} (\pi_M + \pi_L + c) \geq (1 - \delta)(-\pi_M) - \delta\pi_H$$

This inequality is equivalent to:

$$\delta \geq \delta_0, \quad \text{where} \quad \delta_0 = \frac{\pi_L - \pi_M + c}{2(\pi_H - \pi_M)} \quad (6)$$

We conclude that the proposed strategy profile is a Nash equilibrium whenever the CB is sufficiently patient. Note that  $\delta_0 < 1$  because  $c < 2\pi_H - \pi_M - \pi_L$ , by assumption. In other words, if the cost  $c$  were sufficiently high, this candidate equilibrium could not be sustained.

Observe that:

$$\frac{\partial \delta_0}{\partial \pi^*} = \frac{(\pi_H - \pi_M) \left( \frac{\partial \pi_L}{\partial \pi^*} - \frac{\partial \pi_M}{\partial \pi^*} + \frac{\partial c}{\partial \pi^*} \right) - (\pi_L - \pi_M + c) \left( \frac{\partial \pi_H}{\partial \pi^*} - \frac{\partial \pi_M}{\partial \pi^*} \right)}{2(\pi_H - \pi_M)^2}$$

Suppose that the following condition is satisfied for every target  $\pi^* \in [0, 1]$ :<sup>8</sup>

$$\frac{-\partial c}{\partial \pi^*} < \left( \frac{\partial \pi_L}{\partial \pi^*} - \frac{\partial \pi_M}{\partial \pi^*} \right) + \frac{(\pi_L - \pi_M + c) \left( \frac{\partial \pi_M}{\partial \pi^*} - \frac{\partial \pi_H}{\partial \pi^*} \right)}{\pi_H - \pi_M} \quad (7)$$

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<sup>8</sup>Note that both sides of inequality (7) are positive.

If inequality (7) holds, then  $\frac{\partial \delta_0}{\partial \pi^*} > 0$ , that is, the higher the target is, the more patient the CB needs to be in order to sustain the described equilibrium. In this case, if the CB is not sufficiently patient meaning that inequality (6) is violated, then, only relatively lower targets are feasible. Since  $\delta_0(\pi^*)$  is an increasing function of the inflation target, we can solve the inequality  $\delta \geq \delta_0(\pi^*)$  for  $\pi^*$ , obtaining an upper bound, denoted  $\bar{\pi}$ . Formally,  $\bar{\pi}$  is the unique solution of the equation  $\delta = \delta_0(\bar{\pi})$ . Hence, the CB maximizes its utility, subject to choosing a sufficiently low target:

$$\text{Max}_{\pi^* \in [0, \bar{\pi}]} \left\{ \frac{-1}{2} (\pi_M + \pi_L + c) \right\}$$

In general, if we do not want to assume condition (7), at least we have that the CB chooses between the outcome path that leads to the action profile  $(1/2, 1)$  being played always, for some target  $\pi^*$  satisfying  $\delta \geq \frac{\pi_L(\pi^*) - \pi_M(\pi^*) + c(\pi^*)}{2\pi_H(\pi^*) - 2\pi_M(\pi^*)}$ , and the outcome path of always  $(D, D)$ . In the former case, the utility is

$$U_1 = \text{Max}_{\{\pi^* | \delta \geq \delta_0(\pi^*)\}} \left\{ \frac{-1}{2} [\pi_M(\pi^*) + \pi_L(\pi^*) + c(\pi^*)] \right\}$$

In the latter case, the CB's utility becomes

$$U_1 = \text{Max}_{\pi^* \in [0, 1]} \{-\pi_H(\pi^*)\}$$

The next proposition summarizes our results so far.

**Proposition 1** (*Best Equilibrium for the Central Bank*)

(i) *If the CB is sufficiently patient, more precisely, if (6) holds, then, the CB can always implement its preferred equilibrium, in which its optimal target is the unique solution of problem (4).*

(ii) *The CB does not need to play the tightest monetary policy in every round of the repeated game to obtain its best equilibrium. The intensity of cooperation,  $x$ , should be only large enough to make market participants' best responses be their part in the prescribed equilibrium, namely  $x \geq 1/2$ .*

### 3.4 Market Participants' Payoffs

Now, we will analyze the utility of market participants. In our model however, by playing  $x$  just above  $1/2$ , the CB forces every market participant to always choose  $C$ . Note that  $u_2(1/2, 1) = 0$ , for any target  $\pi^*$ . Therefore,  $U_j = 0$ , for every market player  $j \in (0, 1)$ .

It turns out that market players could obtain higher equilibrium payoffs if they were long-run players with sufficiently high discount factor. According to the folk theorem, a long-run player 2, having the same discount factor that player 1 has, could obtain a utility arbitrarily close to  $+g$  if the common discount factor were sufficiently close to 1. To see this point, observe that the minmax payoff of player 1 is  $-\pi_H$ . Hence, the payoff profile  $(-\pi_H, +g)$  is in the closure of the feasible and individually rational set of payoff profiles.<sup>9</sup>

## 4 Inflation Bias

In the equilibrium of the repeated game, the inflation at every period  $t$ , and consequently the accumulated inflation  $\Pi$ , are given by:

$$\frac{\pi_M + \pi_L}{2}$$

Note however, that according to (4), the CB chooses a target  $\pi^*$  such that:

$$\pi^* = \arg \min_{\pi^* \in [0, 1]} \{ \pi_M + \pi_L + c \}$$

Hence, there is an inflation bias in the *choice of the target*. The target selected by the CB will be higher than the one that minimizes inflation. If the socially optimal

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<sup>9</sup>There are many versions of the folk theorem, for instance see Fudenberg and Maskin (1986). In fact, this kind of payoff can be obtained in a subgame perfect equilibrium, as long as the common discount factor is sufficiently close to one.

target is the one that minimizes inflation,<sup>10</sup> the classic solution for this problem is to have another member of the government choosing the target, in order to solve:

$$\underset{\pi^* \in [0,1]}{\text{Min}} \{ \pi_M + \pi_L \},$$

Since both  $\pi_M$  and  $\pi_L$  are increasing in the target, the socially optimal target will be  $\pi^* = 0$ . Of course, the branch of the government picking the target must have the correct incentives to do it without any bias. Another potential problem is that the CB remains responsible for implementing the target  $\pi^* = 0$ . If the CB is sufficiently patient in the sense that  $\delta \geq \frac{\pi_L(0) - \pi_M(0) + c(0)}{2\pi_H(0) - 2\pi_M(0)}$ , the outcome path is the perpetual play of the action profile  $(1/2, 1)$ , inflation becomes  $\Pi = \frac{\pi_M(0) + \pi_L(0)}{2}$ , and the first best solution is implemented. Otherwise, when  $\delta < \frac{\pi_L(0) - \pi_M(0) + c(0)}{2\pi_H(0) - 2\pi_M(0)}$ , the outcome path will be the repeated play of  $(D, D)$ , and inflation becomes  $\Pi = \pi_H(0)$ .

In this case, there are two possibilities for decreasing inflation. First, it could be better to choose the minimum possible target  $\pi^*$  such that  $\delta \geq \frac{\pi_L(\pi^*) - \pi_M(\pi^*) + c(\pi^*)}{2\pi_H(\pi^*) - 2\pi_M(\pi^*)}$ , in order to implement the perpetual play of  $(1/2, 1)$  as the equilibrium of the repeated game. Another alternative is to allow the CB to choose the target.

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<sup>10</sup>In some models of the literature, the output gap is also considered. However, microfounded models show that the weight of the output gap term in the respective loss function is so small that we may ignore it without any serious consequence. For instance, see table 5 of Giannone and Woodford (2003), pp. 52.

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